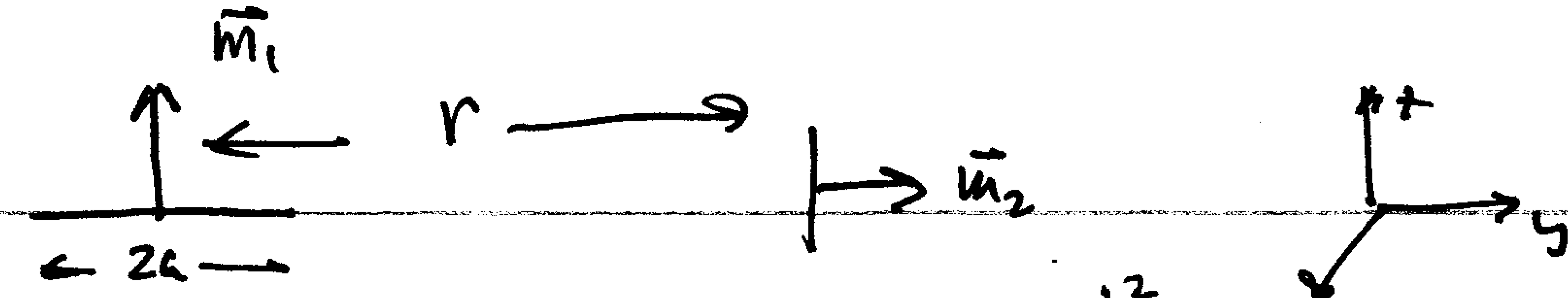
PHYS 321
Assignment 5
Due Monday, April 23, 2018

Read Chapters 6, 7

Problems of Chapter 6: 1, 3b, 12, 16, 17, 23a, 24

6.1)



$m_2 = I b^2$

$m_1 = I \pi a^2$

a) $\vec{N}_2 = \vec{m}_2 \times \vec{B}_1$

$$\vec{B}_1 = \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[3(\vec{m}_1 \cdot \hat{r}) \hat{r} - \vec{m}_1 \right]$$

For co-ordinate system shown

$$\vec{m}_2 = m_2 \hat{y} \qquad \vec{m}_1 = m_1 \hat{z} \qquad \vec{r} = r\hat{y}$$

$$\vec{B}_1 = \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[3(0) - m_1 \hat{z} \right]$$

$$= -\frac{\mu_0}{4\pi} \frac{m_1}{r^3} \hat{z}$$

$$\vec{N}_2 = m_2 \hat{y} \times \vec{B}_1 = -\frac{m_1 m_2 \mu_0}{4\pi r^3} \hat{y} \times \hat{z}$$

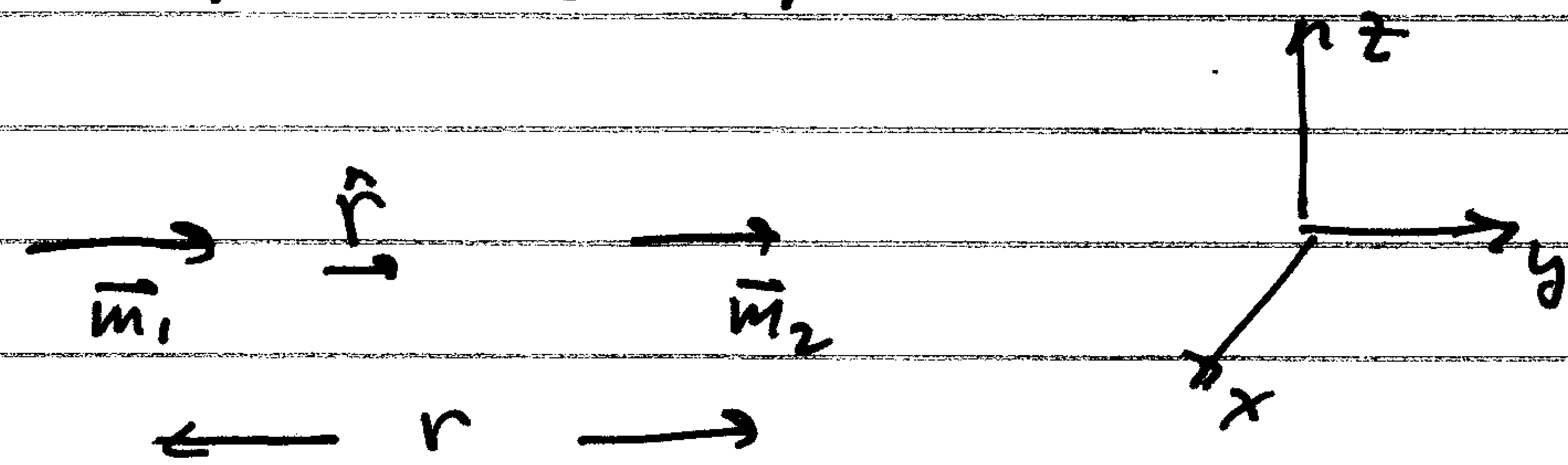
$$= -\frac{m_1 m_2 \mu_0}{4\pi r^3} \hat{x}$$

$$= -\frac{(I \pi a^2)(I b^2) \mu_0}{4\pi r^3} \hat{x}$$

$$= -\frac{(I a b)^2 \mu_0}{4 r^3} \hat{x}$$

b) $U = -\vec{m} \cdot \vec{B}$ the potential energy is minimum when $\vec{m}$ is aligned with $\vec{B}$. So $\vec{m}_2$ will be along the negative z-direction

6.36 $\vec{F} = \nabla(\vec{m} \cdot \vec{B})$



$$\vec{B}_1 = \frac{\mu_0}{4\pi r^3}\left[3(\vec{m}_1 \cdot \hat{r})\hat{r} - \vec{m}_1\right]$$

$$\vec{m}_1 = m_1 \hat{y} \qquad \hat{r} = r\hat{y} \qquad \vec{m}_2 = m_2 \hat{y}$$

$$\vec{B}_1 = \frac{\mu_0}{4\pi r^3}\left[(3)\, m_1 \hat{y} - m_1 \hat{y}\right]$$

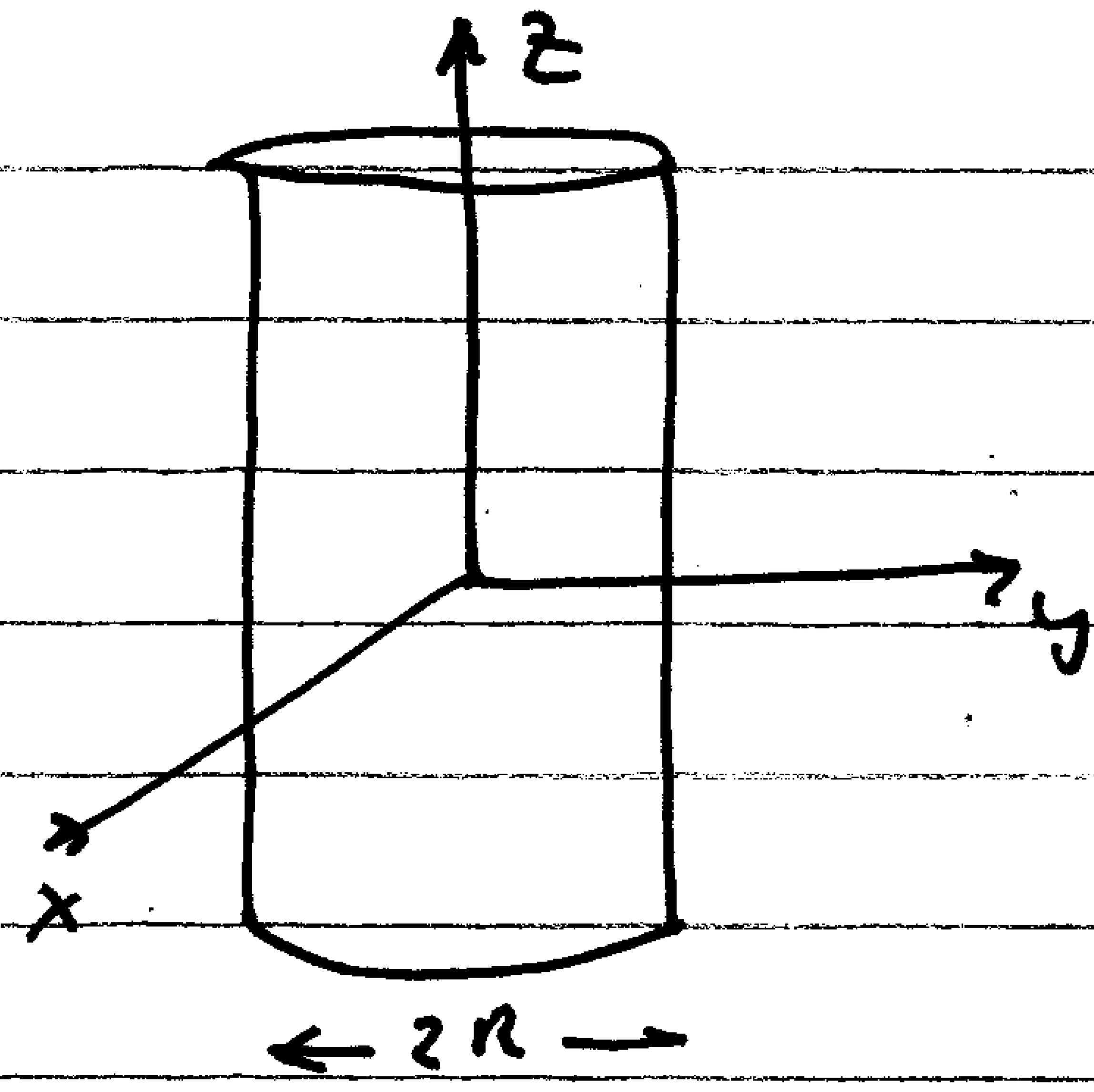
$$= \frac{\mu_0 m_1}{2\pi r^3}\hat{y}$$

$$\vec{m}_2 \cdot \vec{B}_1 = \frac{\mu_0 m_1 m_2}{2\pi r^3}$$

$$\vec{F}_{2-1} = \nabla\left(\vec{m}_2 \cdot \vec{B}_1\right) = \frac{-3\mu_0 m_1 m_2}{2\pi r^4}\hat{r}$$

= force on m_2 due to m_1, attractive

6.12)



$\vec{M} = k s \hat{z}$

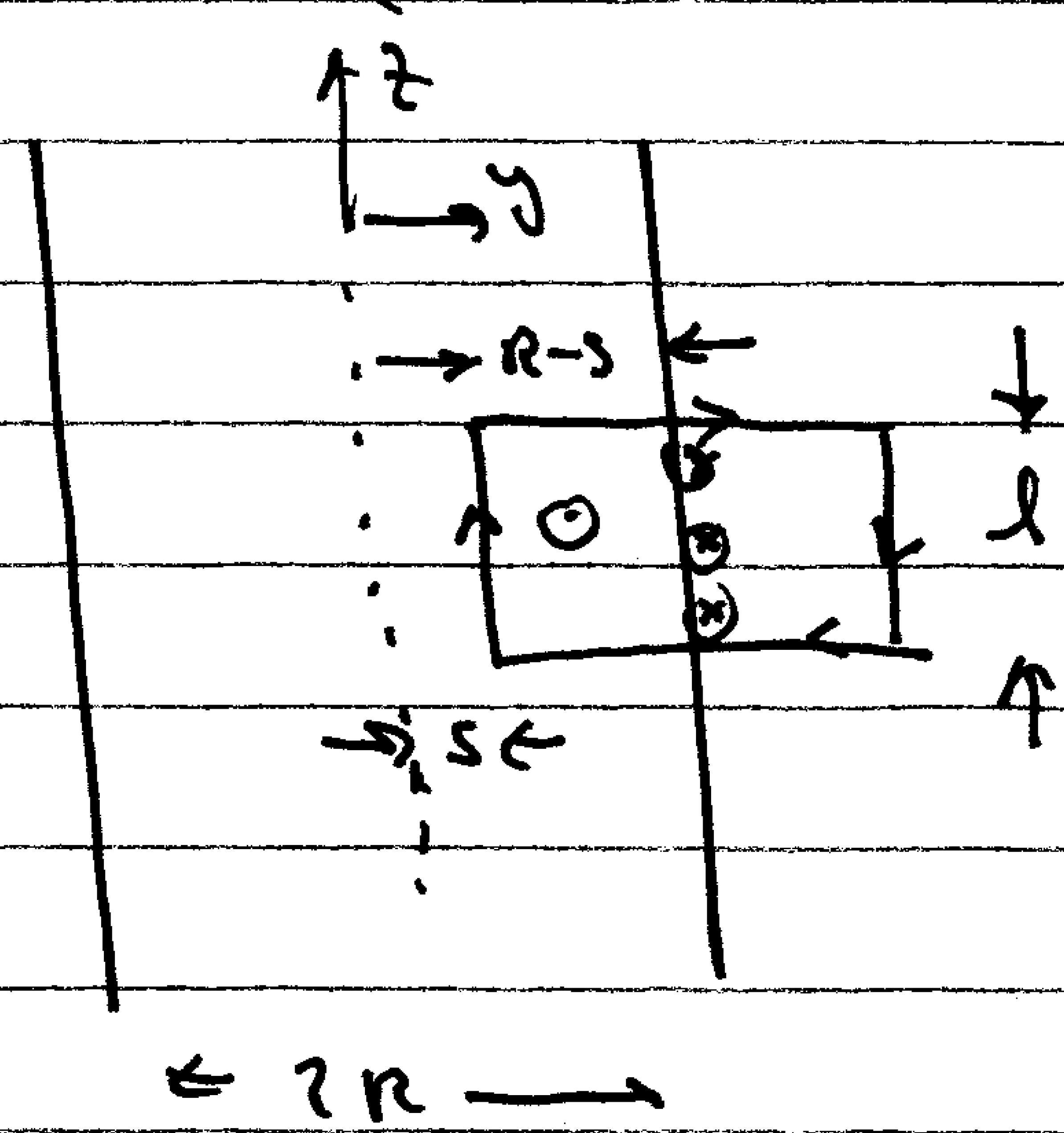
a) $\vec{K}_b = \vec{M} \times \hat{n} = kR\, \hat{z} \times \hat{s} = kR\, \hat{\varphi}$

$$\vec{J}_b = \nabla \times \vec{M} = \frac{1}{s}\cancel{\frac{\partial M_z}{\partial \varphi}}\,\hat{s} - \frac{\partial M_z}{\partial s}\,\hat{\varphi}$$

$$= -k\,\hat{\varphi}$$

Note that for $r > s$ $\vec{H} = 0$, $\vec{B} = 0$, $\vec{M} = 0$

Draw an Amperian loop in the z-y plane for convenience



Note $\vec{K}$ is ⊗
$\vec{J}$ is ⊙

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I.$

For RHS $\oint \vec{B}\cdot d\vec{l} = B\,l$

For LHS we get two contributions to I.

the surface $I_{surface} = K_b\,l = kR\,l$

the volume $I_{vol.} = l(R-s)J_b = l(R-s)(-k)$

$$B\,l = \left[-l(R-s)k + kR\,l\right]\mu_0$$

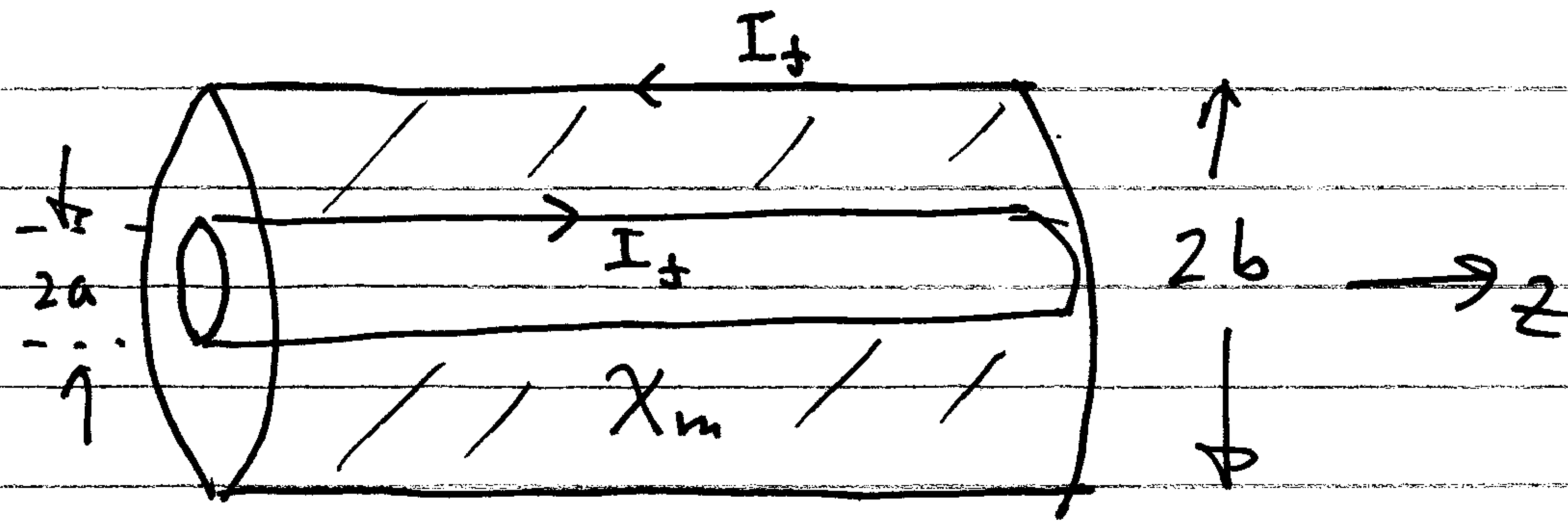
$$= \mu_0\, s\, k\, l$$

$$\vec{B} = \mu_0\, s\, k\,\hat{z}$$

b) $\vec{B} = \mu_0(\vec{H} + \vec{M})$ $\quad \vec{H} = 0 \Rightarrow \vec{B} = \mu_0 \vec{M}$

$$\vec{B} = \mu_0\, k s\,\hat{z}$$

6.16



a) $\nabla \times \vec{H} = \vec{J}_f \qquad \oint \vec{H} \cdot d\vec{l} = I_f$

$$H\, 2\pi s = I_f \qquad \vec{H} = \frac{I_f}{2\pi s}\hat{\varphi} \qquad a < s < b$$

$$s > b \qquad H = 0, \quad B = 0$$

For a linear medium $\vec{B} = \mu \vec{H} = \mu_0(1+\chi_m)\vec{H}$

$$\vec{B} = \mu_0(1+\chi_m)\frac{I_f}{2\pi s}\hat{\varphi} \qquad a < s < b$$

b) $\vec{M} = \chi_m \vec{H} = \dfrac{\chi_m I_f}{2\pi s}\hat{\varphi} \qquad a < s < b$

$$\vec{K}_b = \vec{M} \times \hat{n} \qquad \hat{n} = \hat{s}$$

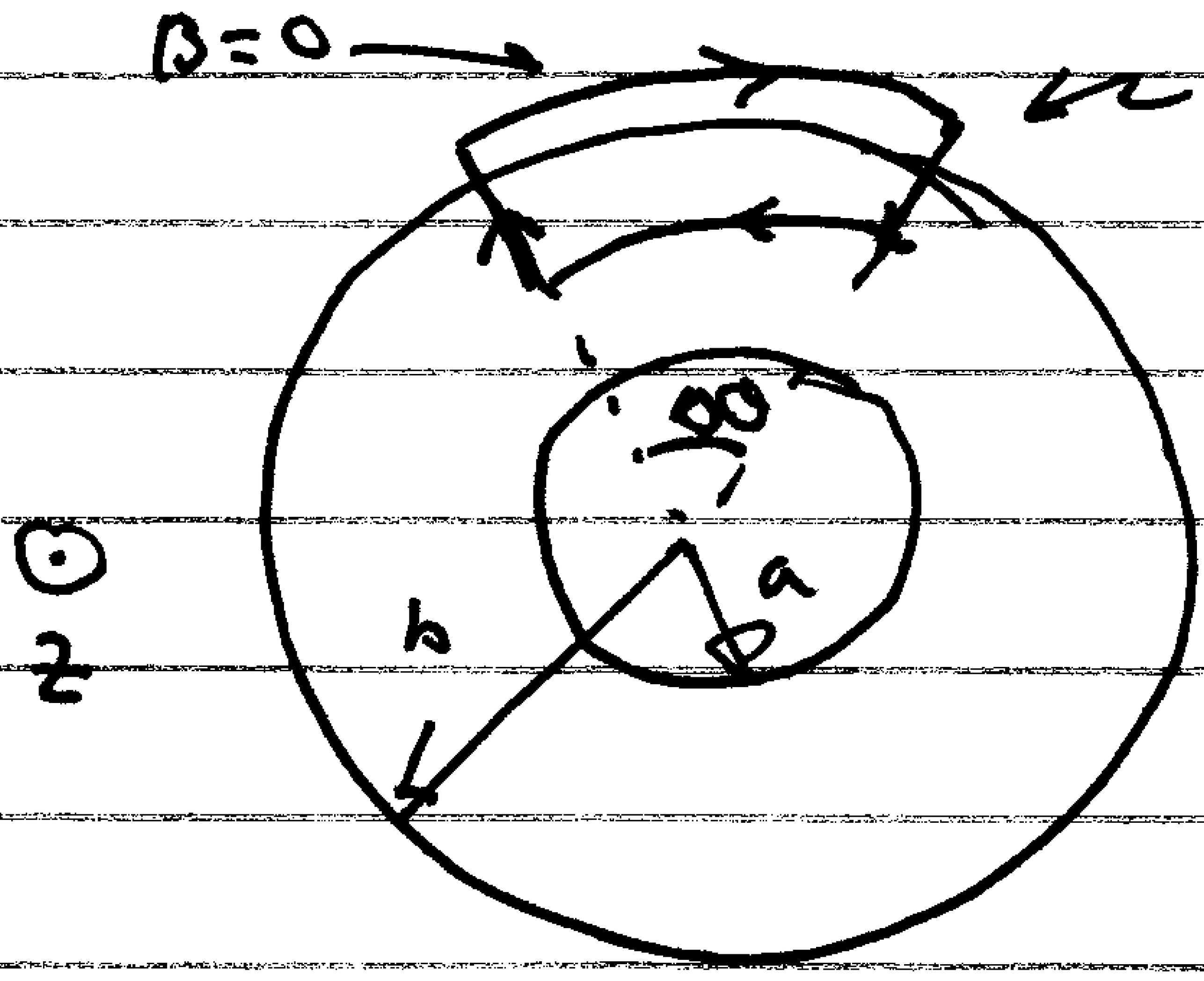
$$\vec{K}_b^{(outer)} = \frac{\chi_m I_f}{2\pi b}\hat{\varphi} \times \hat{s}$$

$$= -\frac{\chi_m I_f}{2\pi b}\hat{z}$$

$$\vec{K}_b^{(inner)} = +\frac{\chi_m I_f}{2\pi a}\hat{z}$$

$$\vec{J}_b = \nabla \times \vec{M} = \frac{1}{s}\frac{\partial}{\partial s}\left(\frac{s\, \chi_m I_f}{2\pi s}\right) = 0$$

total "bound" current $= K_b^{(\text{outer})} + K_b^{(\text{inner})} \neq 0$



Two contributions to

$$\oint \bar{B}\cdot d\bar{l} = \mu_0 I_{\cancel{tot}}$$

from I_f and $K_b^{(\text{outer})}$

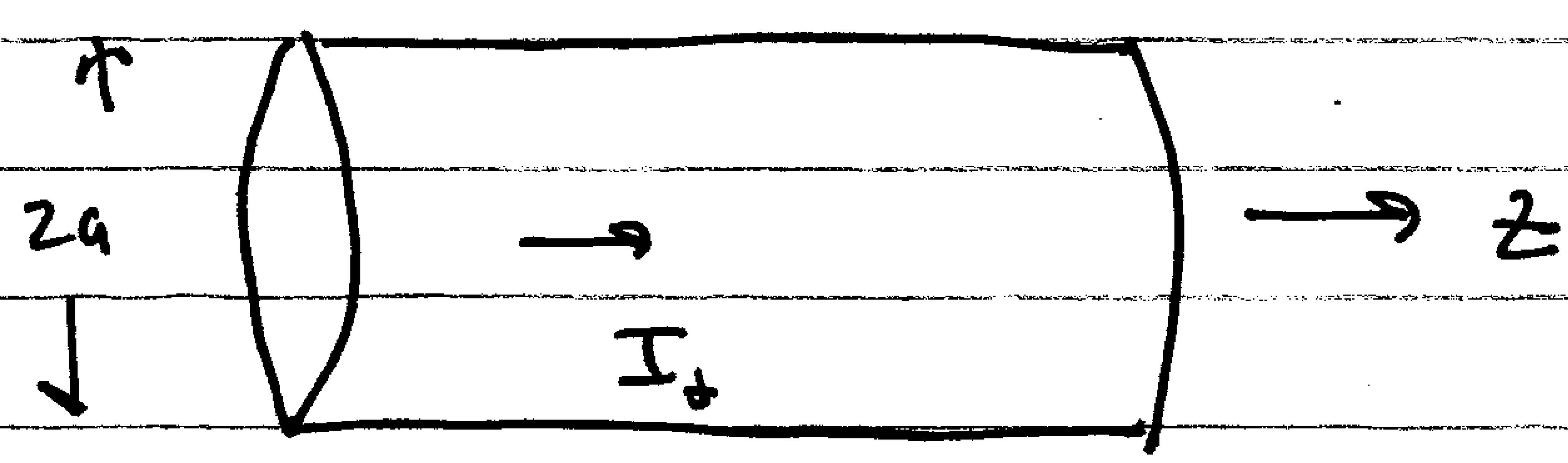
$$\oint \bar{B}\cdot d\bar{l} = \mu_0\left(I_f \frac{b\Delta\theta}{2\pi b} + K_b\, b\,\Delta\theta\right)$$

$$= B\, s\, \Delta\theta = \mu_0\left[I_f \frac{\Delta\theta}{2\pi} + \left(\frac{\chi_m I_f}{2\pi b}\right) b\,\Delta\theta\right]$$

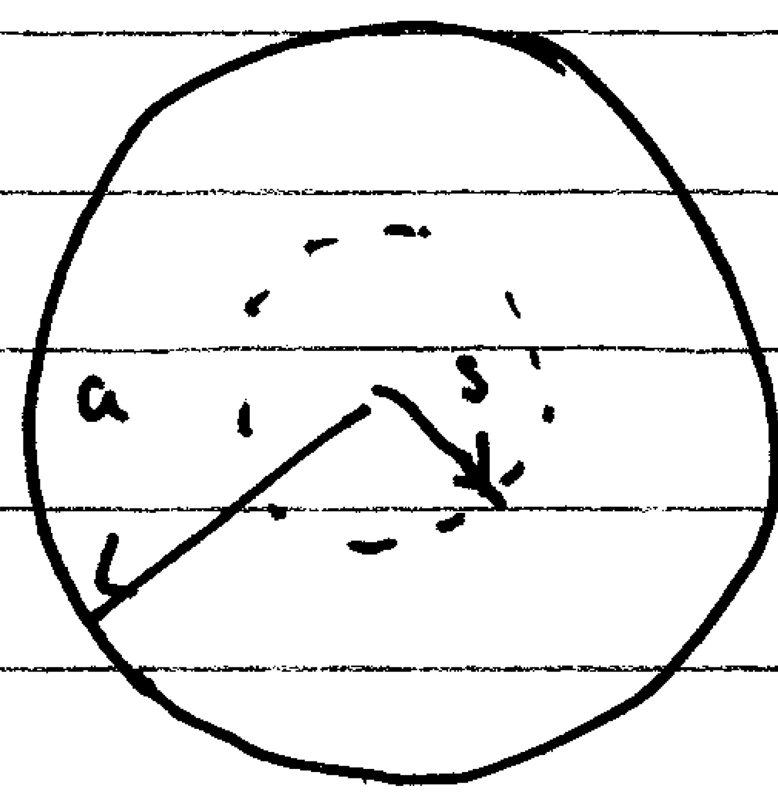
$$B = \frac{\mu_0 I_f (1+\chi_m)}{2\pi s}$$

Note: both I_f and K_b are ⊗ along $-z$ direction in the diagram above.

6-17)



$$\vec{B} = \mu \vec{H} = \mu_0(1+\chi_m)\vec{H} \cdot \quad \vec{M} = \chi_m \vec{H}$$



$$\oint \vec{H}\cdot d\vec{l} = I_f$$

$$2\pi s H = \left(\frac{\pi s^2}{\pi a^2}\right) I_f$$

$$\vec{H} = \frac{I s}{2\pi a^2}\,\hat{\varphi} \quad s < a \qquad \vec{B} = \frac{\mu_0(1+\chi_m) I s}{2\pi a^2}\,\hat{\varphi} \quad s<a$$

$$\vec{H} = \frac{I}{2\pi s}\,\hat{\varphi} \quad s > a \qquad \vec{B} = \frac{\mu_0 I}{2\pi s}\,\hat{\varphi} \quad s > a$$

$$\vec{M} = \frac{I\chi_m s}{2\pi a^2}\,\hat{\varphi} \qquad \vec{J}_b = \nabla\times\vec{M} = \frac{I\chi_m}{2\pi a^2}\,\frac{1}{s}\,\frac{\partial}{\partial s}(s^2)\,\hat{z}$$

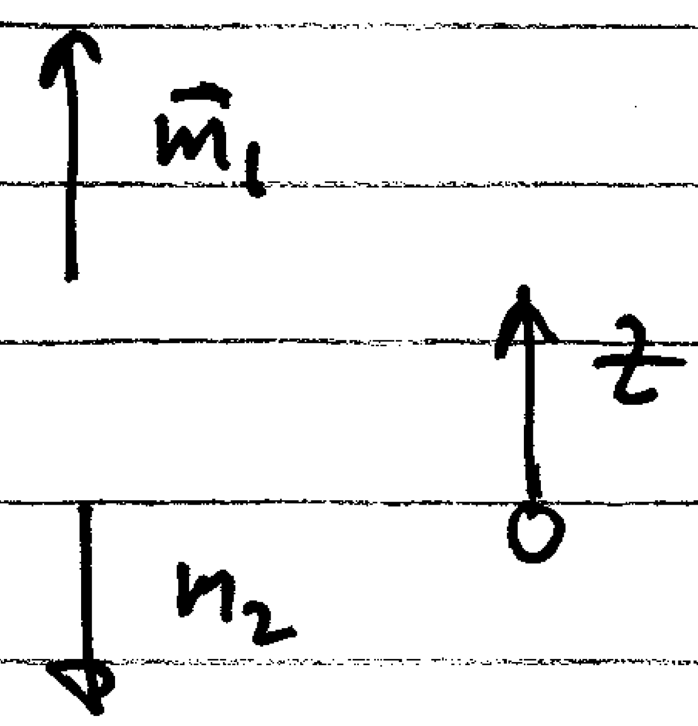
$$\vec{J}_b = \frac{I\chi_m}{\pi a^2}\,\hat{z}$$

$$\vec{K}_b = \vec{M}\times\hat{n} = \frac{I\chi_m a}{2\pi a^2}\,\hat{\varphi}\times\hat{s} = -\frac{I\chi_m}{2\pi a}\,\hat{z}$$

$$\text{Total } I_b = J_b\,\pi a^2 + K_b\,2\pi a$$

$$= I\chi_m - I\chi_m = 0$$

6.23 a)



$\vec{m}_1 = m_1 \hat{z}$

$\vec{m}_2 = -m_2 \hat{z}$

$$\vec{B}_2 = \frac{\mu_0}{4\pi z^3}\left[3(\vec{m}_2 \cdot \hat{z})\hat{z} - \vec{m}_2\right]$$

$$= \frac{\mu_0}{4\pi z^3}\left[-3m_2 + m_2\right]\hat{z}$$

$$= \frac{-2\mu_0 m_2}{4\pi z^3}\hat{z}$$

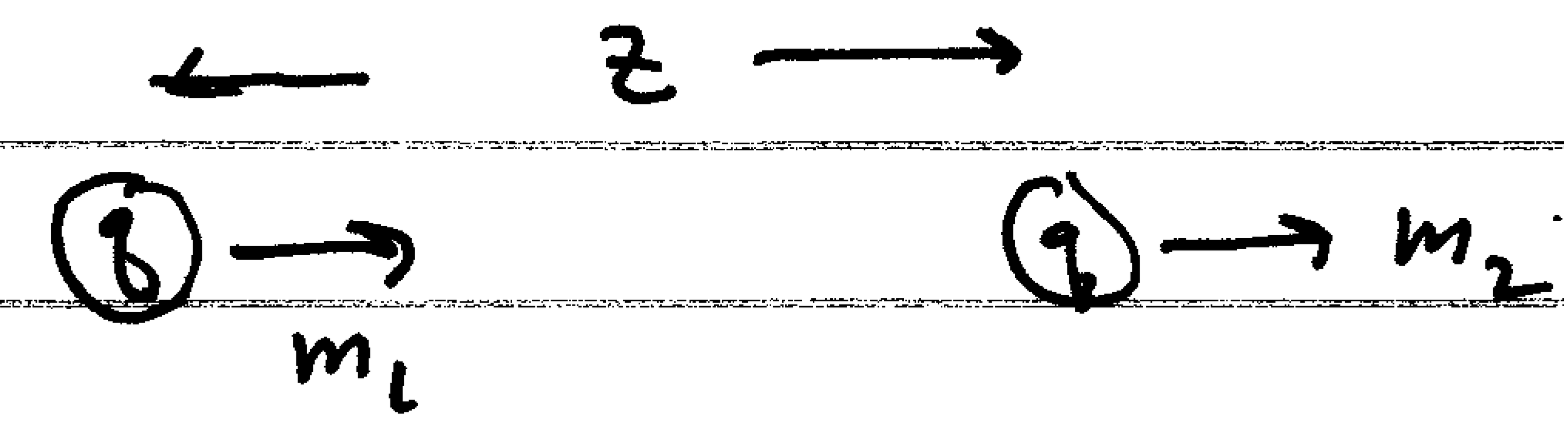
$$\vec{m}_1 \cdot \vec{B}_2 = \frac{-2\mu_0 m_1 m_2}{4\pi z^3}$$

$$\nabla(\vec{m}_1 \cdot \vec{B}_2) = \frac{3\mu_0 m_1 m_2}{2\pi z^4}\hat{z}$$

Let $m_1 = m_2 = m$ mass of dipole $= m_d$

$$\frac{3\mu_0 m^2}{2\pi z^4} = m_d g \qquad z = \left(\frac{3}{2\pi}\frac{m^2\mu_0}{g\, m_d}\right)^{1/4}$$

6.24)



a) From problem 6.36 we know the magnetic force is attractive and equal to

$$F_{dipole} = \frac{3\mu_0 m_1 m_2}{2\pi z^4} \qquad F_{elec} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{z^2}$$

(repulsive)

Let $m_1 = m_2$ $\quad F_{dipole} = F_{elec}$

$$\frac{3\mu_0 m^2}{2\pi z^4} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{z^2}$$

$$z^2 = \frac{3\mu_0 m^2}{2\pi}\left(\frac{4\pi\varepsilon_0}{q^2}\right)$$

$$z_0 = \frac{m}{q}\sqrt{6\mu_0\varepsilon_0}$$ the equilibrium distance

$$= \frac{m\sqrt{6}}{qc} \quad \text{since} \quad c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

b) Let $q = 1.6\times10^{-19}\,C$, $m = \frac{q\hbar}{2m_e} = \frac{qh}{4\pi m_e}$ (Bohr magneton)

$$m = \frac{(1.6\times10^{-19}C)(6.6\times10^{-34}Js)}{(4\pi)(9.1\times10^{-31}kg)} = 9.2\times10^{-24}\ A\,m^2$$

$$z_0 = \frac{(9.2\times10^{-24}\,\mathrm{A\,m^2})\sqrt{6}}{(1.6\times10^{-19}\,\mathrm{C})(3.0\times10^{8}\,\mathrm{m/s})}$$

$= 4.7\times10^{-13}\,\mathrm{m}$ which is much smaller than a hydrogen atom $\approx a_0 = 5.0\times10^{-11}\,\mathrm{m}$, the Bohr radius.